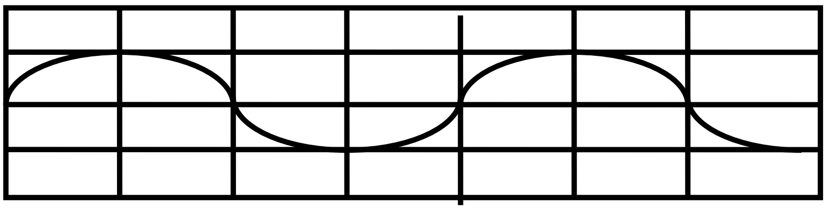
04 Oscillations and waves review answers

Part A: SHM:

1. The defining equation for SHM is $a = -\omega^2 x$.
    a. Explain the terms of this equation and how it relates to the conditions necessary for SHM.
    Acceleration = - (angular velocity)^2 x displacement from equilibrium. The condition necessary for SHM are that the restoring force is proportional to the displacement from the equilibrium and in the opposite direction. Since acceleration is proportional to force this equation is equivalent to F=-kx where k is a positive constant.
    b. For an oscillation of 8Hz calculate the value of T and ω
    $T = 1/f = 0.125s$, $\omega = 2\pi/T = 50$ rad s^{-1}
2. A student connects a signal generator output to a cathode ray oscilloscope input. He gets the following trace on the oscilloscope using a time base of 5 ms per division and a voltage scale of 2 Volts per division.



    a. Calculate the period, frequency, angular frequency and amplitude of the signal.
    Period = time for one cycle = 4 squares = 20ms, Frequency = 1/T = 50Hz, Angular frequency $\omega = 2\pi f = 310$rads^{-1}. (two s.f.), Amplitude = 1 square = 2V
    b. Write a formula for x that describes this signal.
    X = 2 sin(310t)
    c. Calculate the maximum rate of change of voltage of the signal.
    Vmax = ω x_o (when x=0) = 610Vs^{-1}
3. The period of oscillations of a mass of m kg on a spring of stiffness constant k is given by:

    $$T = 2\pi\sqrt{(m/k)}$$

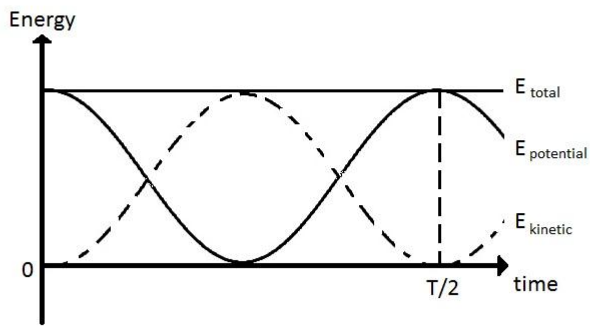
    a) Calculate the period of oscillation of mass of 4 kg on a spring of stiffness constant 200 Nm^{-1}?
    T = 0.89s ($\omega = 2\pi/T = 7.1s^{-1}$)
    b) If the amplitude of the oscillation is 0.05m calculate the kinetic, potential and total energies of the system when the mass is 0.03m from equilibrium.

    $$E_K = \tfrac{1}{2}m\omega^2(x_0^{\,2} - x^2)$$

    $$E_{K(max)} = \tfrac{1}{2}m\omega^2 x_0^{\,2}$$

    $$E_T = \tfrac{1}{2}m\omega^2 x_0^{\,2}$$

    $E_k = 0.5x4x7.1^2(0.05^2-0.03^2) = 0.16J$
    Total energy = $0.5x4x7.1^2x0.05^2$=0.25J
    Ep = 0.25 - 0.16 =0.09J
    c) Sketch a graph showing the changing values of kinetic, potential and total over time.

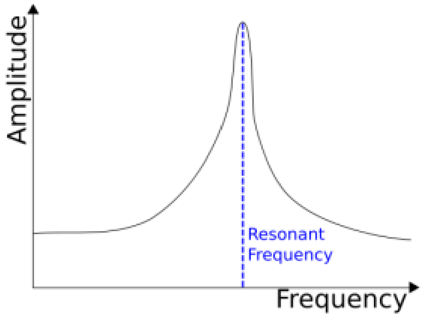


    d) Explain the effect of air resistance and friction in the mass-spring system.
    Air resistance and friction cause damping and so the SHM will gradually reduce in amplitude

Part B: Forced oscillations and resonance:

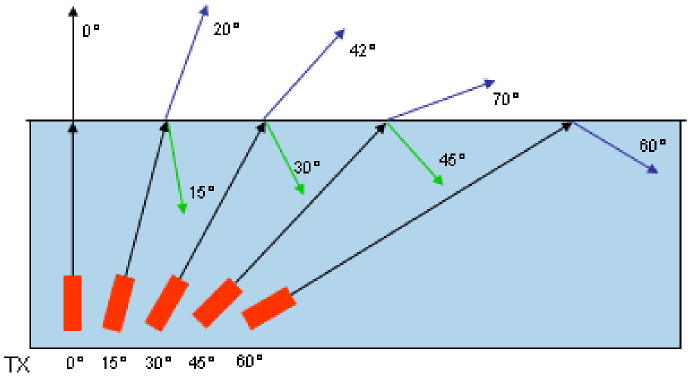
4. The damping you hopefully described in question 3d) is light damping. Explain this term and the meaning of the term critical damping.
Light damping means the system will oscillate several times before stopping, critical damping the system will stop moving in the minimum time.

5. Explain the term resonance and describe two examples of where the phenomenon is useful and one where it is not.
   Resonance occurs when a force oscillates a system at its natural frequency.
   …useful in the ear when hairs resonate at their particular frequency and allow us to detect pitch.
   …useful when pushing a child on a swing. Not useful when vibrations build up in machinery or structures



6. The graph opposite shows the response of a system to changing the frequency of a forcing vibration. Sketch the response to the forcing vibration of:
   a. A system with less damping. **Higher peak**
   b. A system with a higher natural frequency.
   Peak moved to the right.

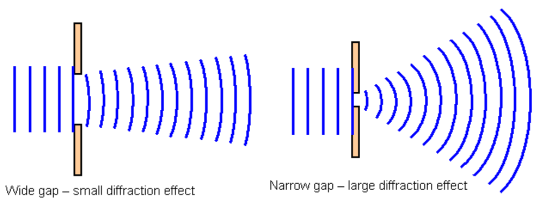
Part C: Wave characteristics and properties

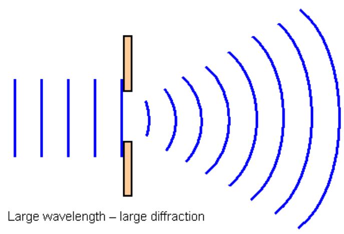


1. In an experiment lasers are used so that 5 light rays are travelling though a glass block as shown opposite.
   a. Redraw the 45^o ray to include a normal line and label the angle of incidence, reflection and refraction.
   Angles should be labeled from the added on normal line.
   b. Calculate the refractive index of the glass block.
   sin(70)/sin(45) = 1.37

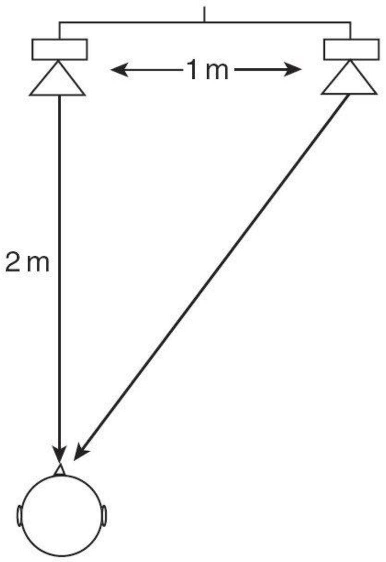
2. A light wave ($c=3.0x10^8$m/s, $f=1x10^{15}$Hz) travels into a glass block of refractive index n=1.4. The incident angle is 20^o. Draw an accurate ray diagram of the refraction that occurs. State also the speed of light and the wavelength of the light in the glass.
   Angle of refraction should be sin^{-1}(sin(20)/1.4) = 14.2^o
   Speed = c/1.4 = 2.14x10^8 ms^{-1}.
   Wavelength = speed / frequency = 2.1x10^{-7}m

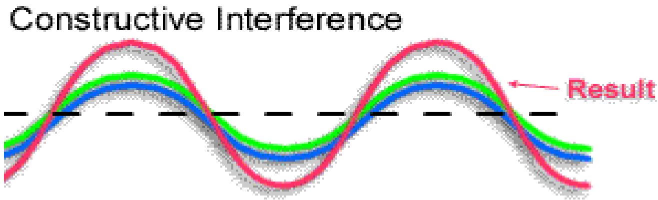
3. The diagrams opposite show diffraction of a wave as it passes through a gap.
   a. Explain why the first picture shows a moderate amount of diffraction. **Wavelength smaller than gap**
   b. Explain why the second two pictures show similar amounts of diffraction.
   Wavelength similar size to gap

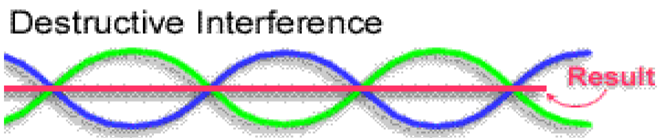




4. Two loudspeakers are positioned as shown below. Pairs of identical notes are played through both loudspeakers simultaneously. The listener notes how loud the sound seems and then the frequency is changed. The listener notices that some notes are louder than others.
   a. Calculate the path difference for the listener from the two speakers. **0.24m**
   b. State the value of the path difference in terms of wavelength that creates destructive interference.
   c. Calculate the wavelength for the lowest pitch note that is quiet. **0.47m**
   d. Using a speed of sound of 330 ms^{-1} calculate the frequency of this note. **750Hz**

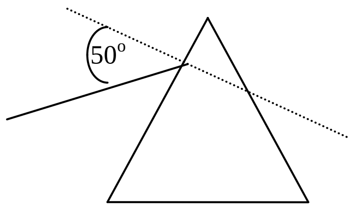






5. A water wave approaches a wall. The water wave has a speed of 2m/s and a frequency of 1Hz.
   (a) What is its wavelength? **2m**
   (b) The wave direction is straight into the wall and the water wave reflects. Explain why there will be places in the water where the wave height will be large and places where it will be small. **The reflected wave will superimpose on the incoming wave, in some places constructively and in others destructively.**

Extension question:



A white light ray is incident on an equilateral glass prism of refractive as shown in the diagram. The refractive index of the glass is 1.40 for red light and 1.42 for violet.

(a) At what angle will the red light leave the block?
(b) At what angle will the violet light leave the block?
(c) Over how many degrees is the visible spectrum dispersed?

For red light:
Sin(r1) = sin(50)/1.4 so r1 = 33.173° by geometry i2=180-120-33.173 = 26.827°
Sin(r2)=sin(26.827)*1.4 so r2 = 39.2°
For blue light:
Sin(r1) = sin(50)/1.42 so r1 = 22.991° by geometry i2=180-120-33.173 = 37.001°
Sin(r2)=sin(37.001)*1.42 so r2 = 77.5°
The real difference in refractive index is much smaller than this so the difference between red and blue calculated here is much exaggerated.